Answer ALL questions. Write your answers in the spaces provided.

1. The line l passes through the points A (3, 1) and B (4, −2).

Find an equation for l.

(3)

Gradient $= \dfrac{y_2 - y_1}{x_2 - x_1}$

$= \dfrac{-2-1}{4-3}$

$= -3$

Equation: $y - y_1 = m(x - x_1)$

Using A(3,1) and m = −3

$\rightarrow y - 1 = -3(x-3)$

$y - 1 = -3x + 9$

$\boxed{y = -3x + 10}$

(Total for Question 1 is 3 marks)

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2. The line l_1 has equation $2x + 4y - 3 = 0$

The line l_2 has equation $y = mx + 7$, where m is a constant.

Given that l_1 and l_2 are perpendicular,

(a) find the value of m.

(2)

The lines l_1 and l_2 meet at the point P.

(b) Find the x coordinate of P.

(2)

a) l_1: $4y = -2x + 3$

$\div 4$: $y = -\frac{2x}{4} + \frac{3}{4}$

so $m_{l_1} = -\frac{1}{2}$

l_1 and l_2 are perpendicular:

$-\frac{1}{2} \times m = -1$

$\therefore m = \frac{-1}{-\frac{1}{2}} = \boxed{2}$

b) $y = \underbrace{2x + 7}_{l_2} = \underbrace{-\frac{1}{2}x + \frac{3}{4}}_{l_1}$

$\Rightarrow \frac{5}{2}x = -\frac{25}{4}$

$\Rightarrow \boxed{x = -\frac{5}{2}}$

3. The curve C has equation

$$y = \frac{k^2}{x} + 1 \qquad x \in \mathbb{R},\ x \neq 0$$

where k is a constant.

(a) Sketch C stating the equation of the horizontal asymptote.

(3)

The line l has equation $y = -2x + 5$

(b) Show that the x coordinate of any point of intersection of l with C is given by a solution of the equation

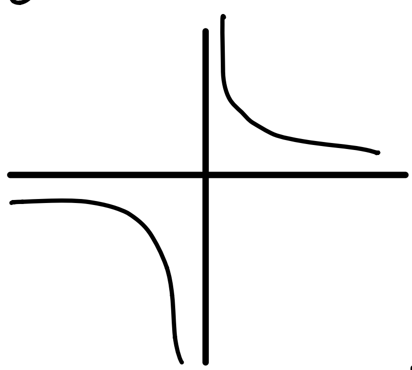
$$2x^2 - 4x + k^2 = 0$$

(2)

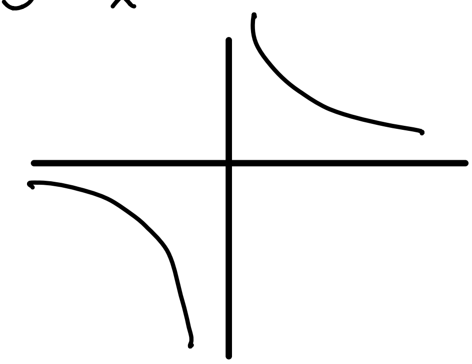
(c) Hence find the exact values of k for which l is a tangent to C.

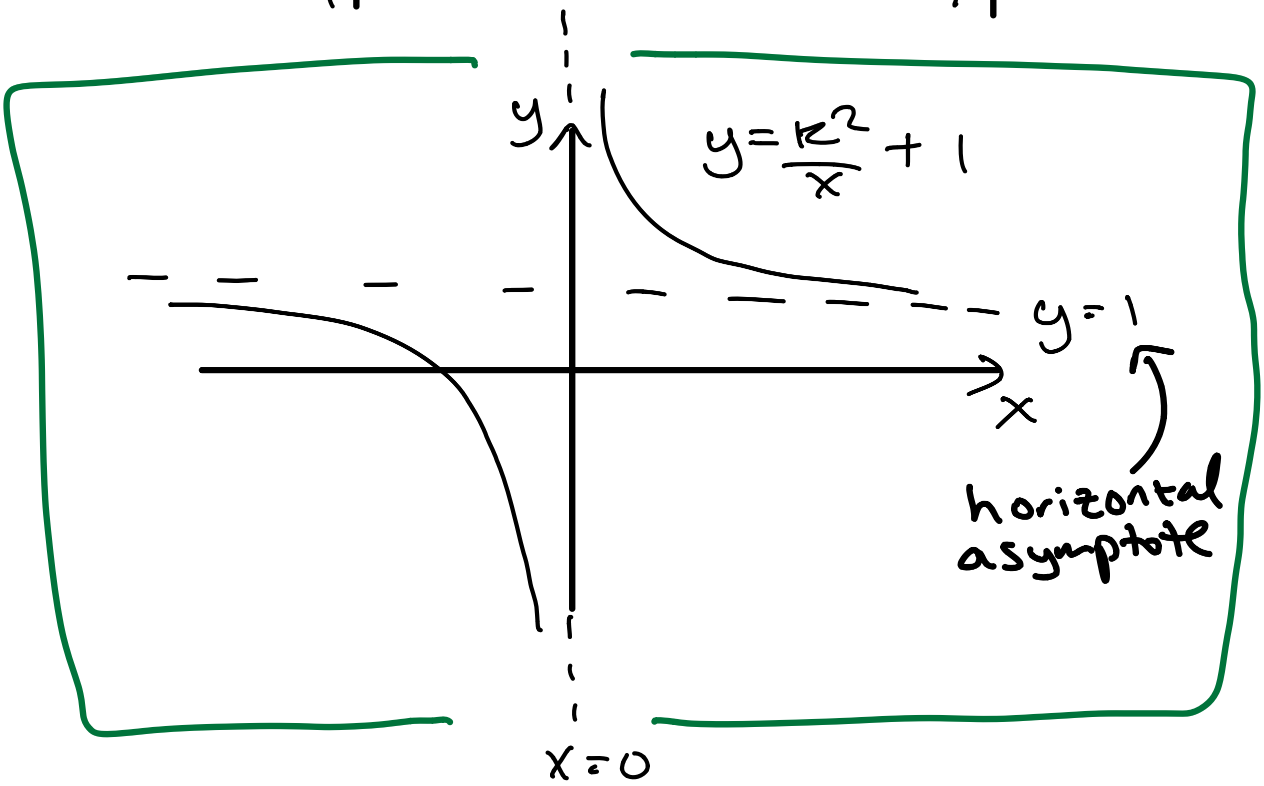
(3)

a) $y = 1/x$



$y = \frac{k^2}{x}$





b) $y = \frac{k^2}{x} + 1$ and $y = -2x + 5$

$\Rightarrow \quad \frac{k^2}{x} + 1 = -2x + 5$

$\Rightarrow \quad k^2 + x = -2x^2 + 5x$

$\Rightarrow \quad 2x^2 - 4x + k^2 = 0$ //

c) tangent; l will only meet C at one point.

So $b^2 - 4ac = 0$

$\Rightarrow \quad (-4)^2 - 4(2)(k^2) = 0$

$\Rightarrow \quad 16 = 8k^2$

$\Rightarrow \quad 2 = k^2$

$\Rightarrow \quad \boxed{k = \pm\sqrt{2}}$

P 5 8 3 5 1 A 0 1 5 4 4

4. A curve has equation

$$y = 2x^3 - 4x + 5$$

Find the equation of the tangent to the curve at the point $P(2, 13)$.

Write your answer in the form $y = mx + c$, where m and c are integers to be found.

Solutions relying on calculator technology are not acceptable.

(5)

tangent : same gradient, same coordinate, one point of intersection $\leftrightarrow$ one root

differentiate $y(x)$: $y = 2x^3 - 4x + 5 \Rightarrow \frac{dy}{dx} = 3 \times 2x^{(3-1)} - 4x^{(1-0)}$

$= 6x^2 - 4$

so gradient @ $P = 6(2)^2 - 4 = 20$

use $y - y_0 = m(x - x_0)$: $y - 13 = 20(x-2)$

$y - 13 = 20x - 40$

$\underline{y = 20x - 27}$

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5.

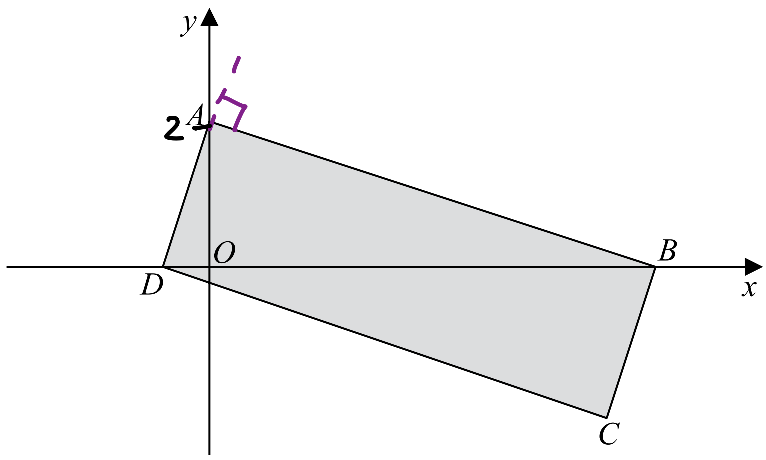


Figure 1

Figure 1 shows a rectangle *ABCD*.

The point *A* lies on the *y*-axis and the points *B* and *D* lie on the *x*-axis as shown in Figure 1.

Given that the straight line through the points *A* and *B* has equation $5y + 2x = 10$

(a) show that the straight line through the points *A* and *D* has equation $2y - 5x = 4$

(4)

(b) find the area of the rectangle *ABCD*.

(3)

a) $5y + 2x = 10$

$y = -\frac{2}{5}x + 2$

$m_{AB} = -\frac{2}{5}$ - 1

$-\frac{2}{5} \times m_{AD} = -1$

$m_{AD} = \frac{5}{2}$ - 1

$y - y_1 = m(x - x_1)$

$y - 2 = \frac{5}{2}(x - 0)$

$y - 2 = \frac{5}{2}x$

$2y - 4 = 5x$

$2y - 5x = 4$ - 1

@ $x = 0$

$5y + 2(0) = 10$

$5y = 10$

$y = 2$

$A(0, 2)$ - 1

b) for B:
@ $y=0$

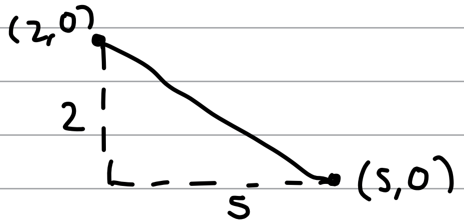
$2x = 10$
$x = 5$

B$(5,0)$

for D:
@ $y=0$

$-5x = 4$
$x = -\frac{4}{5}$

D$(-\frac{4}{5}, 0)$



$AD = \sqrt{(-\frac{4}{5})^2 + 2^2}$
$= \frac{2\sqrt{29}}{5}$

$AB = \sqrt{2^2 + 5^2}$
$= \sqrt{29}$ - 1

Area $= \sqrt{29} \times \frac{2\sqrt{29}}{5}$ - 1

$= 11.6$ - 1

6. A small factory makes bars of soap.

On any day, the total cost to the factory, £y, of making x bars of soap is modelled to be the sum of two separate elements:

- a fixed cost
- a cost that is proportional to the number of bars of soap that are made that day

(a) Write down a general equation linking y with x, for this model.

(1)

The bars of soap are sold for £2 each.

On a day when 800 bars of soap are made and sold, the factory makes a profit of £500

On a day when 300 bars of soap are made and sold, the factory makes a loss of £80

Using the above information,

(b) show that $y = 0.84x + 428$

(3)

(c) With reference to the model, interpret the significance of the value 0.84 in the equation.

(1)

Assuming that each bar of soap is sold on the day it is made,

(d) find the least number of bars of soap that must be made on any given day for the factory to make a profit that day.

(2)

a)

$y = \underbrace{k}_{\text{Proportional Constant.}}x + c \}$ fixed ①

Fixed Cost => Constant

proportional Cost => related to number of bars made

b) i) $y = 2(800) - 500 = 1100 \Rightarrow (x, y) = (800, 1100)$ * $y = kx + c$ *

ii) $y = 2(300) + 80 = 680 \Rightarrow (x, y) = (300, 680)$ ①

$\Rightarrow$ i) $1100 = 800k + c$ and for ii) $680 = 300k + c$

$c = 1100 - 800k$ and $c = 680 - 300k$

$\Rightarrow 1100 - 800k = 680 - 300k \Rightarrow 500k = 420 \Rightarrow k = 0.84$

$\Rightarrow c = 1100 - 800(0.84) = 428 = c$ ②

$\Rightarrow y = 0.84x + 428$ as required. ③

c) $y = 0.84x + 428$

$\Rightarrow$ 0.84 is the cost of making each additional / extra bar of soap. (1)

d) letting n be the least number of bars required to make a profit.

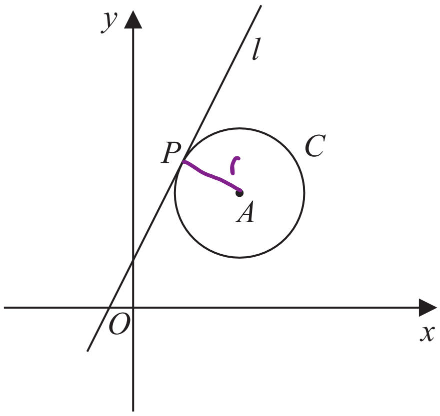
Then $2n = 0.84n + 428$ (1)

$\Rightarrow n = 0.42n + 214 \Rightarrow 0.58n = 214$

$\Rightarrow n = 368.965...$

$\Rightarrow n = 369$ bars (1)

7.



r = radius of C

Figure 3

The circle C has centre A with coordinates $(7, 5)$.

The line l, with equation $y = 2x + 1$, is the tangent to C at the point P, as shown in Figure 3.

$\hookrightarrow m_t = 2$

(a) Show that an equation of the line PA is $2y + x = 17$

(3)

(b) Find an equation for C.

(4)

The line with equation $y = 2x + k$, $k \neq 1$ is also a tangent to C.

(c) Find the value of the constant k.

(3)

a) m_t = tangent gradient. m_r = radius gradient.

for perpendicular lines, $m_1 m_2 = -1$

$m_t \times m_r = -1$

$2 \times m_r = -1$

$m_r = -\frac{1}{2}$ ✓

$y - y_1 = m(x - x_1)$

(x_1, y_1) is a point on the line

$x_1 = 7 \quad y_1 = 5$

$y - 5 = -\frac{1}{2}(x - 7)$ ✓

$2y - 10 = -(x - 7)$

$2y - 10 = -x + 7$

$2y + x = 17$ as required. ✓

b)

PA : $2y + x = 17$ $l: y = 2x+1$ $A(7,5)$

$(x-a)^2 + (y-b)^2 = r^2$

$(x-7)^2 + (y-5)^2 = r^2$

$2(2x+1) + x = 17$

$4x + 2 + x = 17$

$5x + 2 = 17$ ✓

$5x = 15 \quad \therefore x = 3 \quad \Rightarrow y = 2(3)+1$

$= 6+1$

$= 7.$

$P = (3, 7)$ ✓

$|PA| = \sqrt{(P_x - A_x)^2 + (P_y - A_y)^2}$

$= \sqrt{(3-7)^2 + (7-5)^2} = \sqrt{16+4} = \sqrt{20}$ ✓

$r = \sqrt{20} \quad \therefore r^2 = 20$

Equation of C is $(x-7)^2 + (y-5)^2 = 20$ ✓

c)

$C: (x-7)^2 + (y-5)^2 = 20 \qquad y = 2x+k$

tangent => solution exist.

$C: x^2 - 14x + 49 + y^2 - 10y + 25 = 20$

$x^2 - 14x + y^2 - 10y + 54 = 0$

$x^2 - 14x + (2x+k)^2 - 10(2x+k) + 54 = 0$

$x^2 - 14x + 4x^2 + 4kx + k^2 - 20x - 10k + 54 = 0$

$5x^2 + (4k-34)x + k^2 - 10k + 54 = 0$ ✓

$ax^2 + bx + c$

tangent => one solution only : $b^2 - 4ac = 0$ ✓

$(4k-34)^2 \underbrace{- 4(5)}_{-20}(k^2 - 10k + 54) = 0$ ✓

$16k^2 - 272k + 1156 - 20k^2 + 200k - 1080 = 0$

$-4k^2 - 72k + 76 = 0$

$k^2 + 18k - 19 = 0$

$(k+19)(k-1) = 0$

$k+19 = 0 \Rightarrow k = -19$

$k-1 = 0 \Rightarrow k = 1$

$k = -19 \text{ or } 1$, but since $k \neq 1$, $\therefore k = -19$ ✓

8. The circle C has equation

$$x^2 + y^2 - 10x + 4y + 11 = 0$$

(a) Find

(i) the coordinates of the centre of C,

(ii) the exact radius of C, giving your answer as a simplified surd.

(4)

The line l has equation $y = 3x + k$ where k is a constant.

Given that l is a tangent to C, $(b^2 - 4ac = 0)$

(b) find the possible values of k, giving your answers as simplified surds.

(5)

(a) $(x-5)^2 - (-5)^2 + (y+2)^2 - (2)^2 + 11 = 0$

half of coefficient x (ie. 10)

half of coefficient y (ie. 4)

1

$(x-5)^2 + (y+2)^2 = 25 + 4 - 11$

$(x-5)^2 + (y+2)^2 = 18$

(a)(i) $(5, -2)$ 1

(ii) $r = \sqrt{18} = 3\sqrt{2}$ 1 1

(b) $x^2 + y^2 - 10x + 4y + 11 = 0$ — 1

$y = 3x + k$ — 2

Substitute 2 into 1

$x^2 + (3x+k)^2 - 10x + 4(3x+k) + 11 = 0$

$x^2 + 9x^2 + 6kx + k^2 - 10x + 12x + 4k + 11 = 0$ 1

$10x^2 + (6k+2)x + 11 + 4k + k^2 = 0$ 1

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$b^2 - 4ac = 0$

$(6k+2)^2 - 4(10)(11+4k+k^2) = 0$ 1

$36k^2 + 24k + 4 - 440 - 160k - 40k^2 = 0$

$-4k^2 - 136k - 436 = 0$

$4k^2 + 136k + 436 = 0$ 1

$\therefore k = -17 \pm 6\sqrt{5}$ 1

7. 9. **In this question you should show all stages of your working.**

Solutions relying entirely on calculator technology are not acceptable.

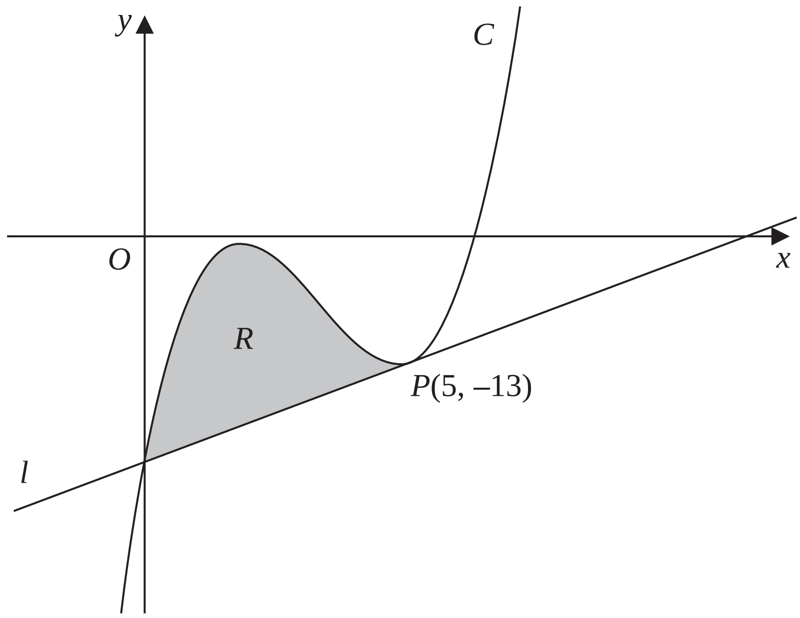

Figure 2

Figure 2 shows a sketch of part of the curve C with equation

$$y = x^3 - 10x^2 + 27x - 23$$

The point $P(5, -13)$ lies on C

The line l is the tangent to C at P

(a) Use differentiation to find the equation of l, giving your answer in the form $y = mx + c$ where m and c are integers to be found.

(4)

(b) Hence verify that l meets C again on the y-axis.

(1)

The finite region R, shown shaded in Figure 2, is bounded by the curve C and the line l.

(c) Use algebraic integration to find the exact area of R.

(4)

a) $y = x^3 - 10x^2 + 27x - 23$ — we know the point $(5, -13)$ is on the line l.

$\frac{dy}{dx} = 3x^2 - 20x + 27$ (1)

when $x = 5$, gradient $\frac{dy}{dx} = 3(5^2) - 20(5) + 27 = 2$ (1)

(1) use formula $y - y_1 = m(x - x_1)$ with point $(5, -13)$

$y - (-13) = 2(x - 5)$

$y + 13 = 2(x - 5)$

$y = 2x - 23$ (1)

b) When $x = 0$ (on the y-axis).
l: $y = 2(0) - 23 = -23$
C: $y = 0^3 - 10(0^2) + 27(0) - 23 = -23$
Both C and l pass through $(0, -23)$, so C meets l again on the y-axis.

difference between area bound by C and area bound by l.

c) $R = \int_0^5 (x^3 - 10x^2 + 27x - 23 - (2x - 23))\, dx$

$R = \left[\frac{x^4}{4} - \frac{10x^3}{3} + \frac{25x^2}{2} \right]_0^5$ (1) for correct integration
(1) for applying correct bounds

$R = \frac{625}{4} - \frac{1250}{3} + \frac{625}{2}$ (1)

$R = \frac{625}{12}$ (1)

to integrate ax^n.

$\int ax^n \, dx = \frac{a}{n+1} x^{n+1} (+c)$

eg $\int 27x \, dx = \frac{27}{1+1} x^{1+1}$

$= \frac{27}{2} x^2$

10. The line l_1 has equation $4y - 3x = 10$

The line l_2 passes through the points $(5, -1)$ and $(-1, 8)$.

Determine, giving full reasons for your answer, whether lines l_1 and l_2 are parallel, perpendicular or neither.

(4)

l_1 $\quad 4y - 3x = 10$

$y = \frac{3x+10}{4}$

$m = \frac{3}{4}$

l_2 $\quad m = \frac{8-(-1)}{-1-5}$

$= -\frac{3}{2}$

$\frac{3}{4} \times \left(-\frac{3}{2}\right) = -\frac{9}{8}$

$-\frac{9}{8} \neq -1$

gradients are not the same

$\therefore$ neither